

CHAPTER 4 - EQUATIONS USING WHOLE NUMBERS

4.1 SOLVING TYPE I EQUATIONS

When solving **Type I** equations algebraically, we use the opposite operation that is shown to determine what value our **variable** (letter) has. Addition is the opposite operation of subtraction, and multiplication is the opposite operation of division. There are 4 kinds of Type I equations that we will be solving and the examples below show how each type is solved.

TYPE 1 (ADDITION)

$$\begin{aligned}x + 7 &= 9 \\x &= 9 - 7 \\x &= 2\end{aligned}$$

TYPE 1 (SUBTRACTION)

$$\begin{aligned}x - 3 &= 4 \\x &= 4 + 3 \\x &= 7\end{aligned}$$

TYPE 1 (MULTIPLICATION)

$$\begin{aligned}7x &= 35 \\7x &= \underline{35} \\7 & \quad 7 \\x &= 5\end{aligned}$$

TYPE 1 (DIVISION)

$$\begin{array}{r}x \\3 \\ \hline 3 \end{array} = 8$$

$\begin{array}{c} \nearrow 8 \\ \searrow 1 \end{array}$

$$x = 24$$



(Note that in each case above we removed the number beside the letter by using the opposite operation. But when we have a division type question we do a special type of opposite operation called **cross-multiplication** as the arrows indicate.)

A. Solve the following showing all steps.

1. $x + 6 = 7$

2. $x - 8 = 12$

3. $3x = 24$

4. $9 + x = 16$

5. $\frac{x}{8} = 21$

6. $7x = 56$

7. $8x = 176$

8. $x + 6 = 53$

9. $x - 15 = 2$

10. $\frac{x}{15} = 2$

11. $12x = 144$

12. $x - 18 = 30$

13. $18x = 72$

14. $6 + x = 6$

15. $\frac{x}{9} = 10$

16. $x + 8 = 12$

17. $x + 12 = 12$

18. $5 + x = 30$

19. $6x = 354$

20. $\frac{x}{7} = 3$

21. $35x = 140$

22. $x + 9 = 13$

23. $x - 9 = 36$

24. $x - 5 = 17$

25. $\frac{x}{9} = 0$

26. $x - 3 = 158$

27. $x + 7 = 42$

28. $x - 9 = 6$

B. Extra Practice. Solve the following.

- | | | | |
|-------------------|------------------------|------------------------|------------------------|
| 1. $n + 7 = 17$ | 2. $6n = 30$ | 3. $x - 59 = 93$ | 4. $\frac{x}{5} = 24$ |
| 5. $n - 9 = 8$ | 6. $2n = 8$ | 7. $x + 23 = 67$ | 8. $\frac{x}{9} = 8$ |
| 9. $n - 5 = 6$ | 10. $4n = 28$ | 11. $x - 7 = 11$ | 12. $\frac{x}{8} = 12$ |
| 13. $n + 3 = 8$ | 14. $\frac{n}{5} = 11$ | 15. $x + 9 = 17$ | 16. $\frac{x}{7} = 13$ |
| 17. $n + 12 = 19$ | 18. $8x = 56$ | 19. $a + 7 = 9$ | 20. $27x = 81$ |
| 21. $n - 5 = 9$ | 22. $\frac{x}{3} = 8$ | 23. $n + 20 = 39$ | 24. $\frac{n}{7} = 7$ |
| 25. $x - 3 = 16$ | 26. $15x = 60$ | 27. $n - 6 = 7$ | 28. $\frac{x}{9} = 7$ |
| 29. $x + 3 = 37$ | 30. $24x = 96$ | 31. $x - 8 = 15$ | 32. $11x = 77$ |
| 33. $x - 1 = 12$ | 34. $6x = 96$ | 35. $n + 3 = 15$ | 36. $\frac{n}{2} = 9$ |
| 37. $x + 1 = 11$ | 38. $3x = 51$ | 39. $\frac{n}{6} = 12$ | 40. $n + 3 = 15$ |

4.2 SOLVING TYPE II EQUATIONS

When solving a **Type II** equation, you will be required to complete two opposite operations. Always do the opposite of any addition or subtraction first, then proceed to do the inverse operation of any multiplication or division, as shown in the examples below.

EXAMPLE #1

$$\begin{aligned}
 5x - 7 &= 8 \\
 5x &= 8 + 7 \\
 5x &= 15 \\
 \frac{5x}{5} &= \frac{15}{5} \\
 x &= 3
 \end{aligned}$$

EXAMPLE #2

$$\begin{aligned}
 \frac{x}{8} + 3 &= 12 \\
 \frac{x}{8} &= 12 - 3 \\
 \frac{x}{8} &= 9 \\
 \frac{x}{8} \cdot 8 &= 9 \cdot 8 \\
 x &= 72
 \end{aligned}$$



A. Solve the following.

1. $3x + 5 = 35$

2. $7x - 6 = 15$

3. $\frac{x}{4} + 2 = 7$

4. $6x - 3 = 45$

5. $\frac{x}{6} - 3 = 6$

6. $2x + 4 = 8$

7. $9x - 3 = 60$

8. $\frac{x}{4} + 5 = 11$

9. $4x - 10 = 30$

10. $\frac{x}{3} - 2 = 8$

11. $6x - 3 = 15$

12. $\frac{x}{3} - 5 = 1$

13. $7x - 3 = 25$

14. $\frac{x}{7} - 2 = 4$

15. $6x + 2 = 14$

16. $12x + 12 = 156$

17. $\frac{x}{5} - 2 = 10$

18. $6x - 4 = 14$

19. $\frac{x}{3} + 2 = 3$

20. $4x + 7 = 95$

21. $6x + 3 = 33$

B. Extra Practice. Solve the following.

1. $5a + 6 = 16$

2. $3x + 1 = 16$

3. $2y + 4 = 10$

4. $4x + 5 = 17$

5. $\frac{x}{6} - 3 = 8$

6. $7x + 5 = 26$

7. $4x - 3 = 17$

8. $\frac{x}{2} + 5 = 9$

9. $5x + 2 = 17$

10. $3x - 8 = 1$

11. $6x + 2 = 44$

12. $\frac{x}{2} - 5 = 7$

13. $8x + 5 = 29$

14. $14x - 38 = 116$

15. $15x + 20 = 95$

16. $7z - 22 = 27$

17. $\frac{x}{5} + 6 = 9$

18. $8x + 2 = 26$

19. $12n - 11 = 133$

20. $4n + 8 = 36$

21. $25x + 14 = 139$

22. $9x + 2 = 65$

23. $2x + 5 = 9$

24. $1x - 5 = 9$

25. $\frac{x}{2} + 9 = 10$

26. $3x + 6 = 15$

27. $20x - 7 = 73$

28. $6x - 25 = 29$

29. $\frac{x}{3} - 8 = 2$

30. $6x + 2 = 2$

31. $\frac{x}{6} + 5 = 12$

32. $3x - 5 = 1$

33. $6x + 6 = 6$

34. $x - 8 = 7$

35. $\frac{x}{7} + 3 = 8$

36. $7x - 5 = 23$

37. $9x + 5 = 32$

38. $14x - 3 = 25$

39. $2x - 5 = 19$

4.3 SOLVING TYPE III EQUATIONS

When you are solving a **Type III** equation, you will first be required to collect and place all the **variables** (letters) on the left side of the equal sign or equation, and all the **constants** (numbers) on the right side of the equal sign or equation.

If an 'x' term is on the right side of the equal sign, bring it over to the left side by using the inverse (opposite) sign. Likewise if a number is being added or subtracted on the left side of an equation, bring it over to the right side by using the inverse (opposite) sign as shown in the two examples below.

EXAMPLE #1

$$\begin{aligned}4x - 3 &= 2x + 13 \\4x - 2x &= 13 + 3 \\2x &= 16 \\ \frac{2x}{2} &= \frac{16}{2} \\x &= 8\end{aligned}$$

EXAMPLE #2

$$\begin{aligned}3x + 5x + 7 - 4x &= 19 + 1x \\3x + 5x - 4x - 1x &= 19 - 7 \\3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\x &= 4\end{aligned}$$



A. Solve the following.

1. $7x + 2 = 5x + 8$
2. $6x + 4x + 2x = 48$
3. $4x + 5x = 81$
4. $3x - 4 = 2x + 9$
5. $7x - 8 = 3x + 4$
6. $2x + 5x = 21$
7. $7x - 3x + 2x = 36$
8. $6x + 2x = 48$
9. $10x - 3 = 5x + 7$
10. $4x - 7 = 3x + 9$
11. $7x - 8 = 2x + 12$
12. $4x - 3x + 5x = 96$
13. $2x + 3x + 4x = 72$
14. $5x + 2 = 3x + 18$
15. $7x - 9 = 3x + 31$
16. $4x - 8 = 3x + 5$
17. $4x - 3x + 2x = 51$
18. $5x + 6 = 3x + 14$
19. $3x - 5 = 2x + 7$
20. $9x + 2 = 5x + 30$
21. $6x + 5x - 2x = 63$
22. $4x + 3x = 7$
23. $5x + 2 = 3x + 2$
24. $8x - 5 = 7x + 10$

B. Extra Practice. Solve the following.

1. $6x + 5 = 2x + 13$ 2. $10x + 75 - 5x = 110$ 3. $4x + 3 + 5x - 2 = 64$

4. $4x + 2 = 3x + 8$ 5. $6x - 4 = 2x + 100$ 6. $4x - 3 = 29$

7. $5x + 7 = 3x + 9$ 8. $9x - 3 = 4x + 22$ 9. $8x - 3 = 2x + 9$

10. $35x - 10 = 32x + 20$ 11. $18x + 3 = 2x + 51$ 12. $4x - 3 = 2x + 9$

13. $8x - 5 = 3x + 5$ 14. $9x - 4 = 2x + 10$ 15. $7x - 5 = 3x + 11$

16. $6x + 5 = 2x + 13$ 17. $3x - 5 + 2x = 75$ 18. $4x + 5x - 8x = 10$

19. $7x - 2 = 3x + 10$ 20. $6x - 3 = 2x + 5$ 21. $16x + 2 = x + 32$

22. $x + x + x + 3 = 15$ 23. $7x + 8 - 2x = 23$ 24. $7x + 2x - 3x + 5 = 47$

25. $6x + 11 + 5x = 66$ 26. $7x + 4 - 5x = 30$ 27. $8x + 5 + 3x + 2 = 95$

28. $6x - 3 = 4x + 9$ 29. $9x - 8 = 3x + 4$ 30. $4x - 3x + 5x - 7 = 11$

4.4 SOLVING TYPE IV EQUATIONS

Identification of a **Type IV** equation is quite simple because these equations contain one or more sets of brackets. Since brackets mean multiplication in mathematics, solving a Type IV equation requires you to first multiply everything on the inside of the set of brackets by the number on the outside of the brackets. The rest of the procedure is the same as if you had a Type II or Type III equation. The examples below show the steps in the procedure.

EXAMPLE #1

$$\begin{aligned} 3(x + 7) &= 39 \\ 3x + 21 &= 39 \\ 3x &= 39 - 21 \\ 3x &= 18 \\ \frac{3x}{3} &= \frac{18}{3} \\ x &= 6 \end{aligned}$$

EXAMPLE #2

$$\begin{aligned} 4(2x - 5) &= 2(3x + 6) \\ 8x - 20 &= 6x + 12 \\ 8x - 6x &= 12 + 20 \\ 2x &= 32 \\ \frac{2x}{2} &= \frac{32}{2} \\ x &= 16 \end{aligned}$$



A. Solve the following.

1. $3(x + 2) = 12$
2. $4(x + 5) = 300$
3. $4(x - 3) = 20$
4. $3(x + 5) = 2(x + 9)$
5. $4(x + 3) = 2(x + 8)$
6. $7(x - 3) = 3(2x + 6)$
7. $6(x - 5) = 30$
8. $4(2x - 3) = 28$
9. $6(5x - 3) = 2(2x + 4)$
10. $4x - 2 = 3(x + 1)$
11. $3(x + 5) = 2(x + 10)$
12. $7(x) - 5 = 2(x + 10)$
13. $3(x - 5) = 2(x + 6)$
14. $7(x - 3) = 5(x + 5)$
15. $3(x - 5) = 2(x + 3)$
16. $2(x + 4) + 5(x + 3) = 100$
17. $4(2x + 2) = 5(x + 7)$
18. $5(3x - 2) = 4(x + 14)$
19. $8(x + 2) = 6(x + 3)$
20. $3(x + 5) + 2(x - 4) = 27$

B. Extra Practice. Solve the following.

1. $4(x + 1) = 12$
2. $5(x + 2) = 30$
3. $7(x - 2) = 21$
4. $5(x - 3) = 4(x + 6)$
5. $3(x + 7) + 2(x - 5) = 36$
6. $4(x + 5) + 2(x - 3) = 20$
7. $4(x + 5) = 3(x + 12)$
8. $2(x - 9) = x + 10$
9. $3(x - 5) = 2(x + 6)$
10. $9(x - 2) = 6(x + 2)$
11. $4(x + 5) + 2(x - 6) = 62$
12. $3(x + 2) = x + 8$
13. $7(2x + 5) = 3(4x + 19)$
14. $(x + 5) = 7$
15. $3(x - 6) = 12$
16. $3(5x - 2) = 9$
17. $2(x + 6) = x + 12$
18. $7(6x + 2) = 7(3x + 5)$
19. $5(x - 6) = 2(x + 3)$
20. $7(x + 3) = 3(x + 7)$
21. $6(x + 3) + 5(x + 3) = 88$
22. $2(x + 5) + 3(x + 6) = 53$
23. $2(x + 2) = (x + 5)$
24. $5(3x + 6) + 2(x + 3) = 70$
25. $x + 5 + 2(x + 3) = 20$
26. $10(x + 2) = 9(x + 6)$

4.5 SOLVING RATIO TYPE EQUATIONS

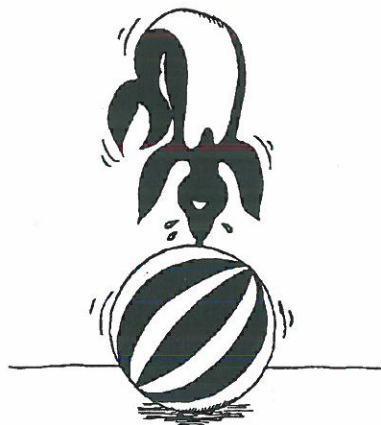
A **Ratio Type** equation is classified as one in which there are two 'fractions', one on either side of the equal sign. To solve these equations we first have to use the procedure known as **cross-multiplication**. Here we multiply either the top left of the equation by the bottom right of the equation or we multiply the top right of the equation by the bottom left of the equation. We place one result on the right side of the equation and the other result on the left side of the equation. We then proceed as we would in a Type II, III or IV question as shown in the examples below.

EXAMPLE #1

$$\begin{aligned} \frac{x}{6} &= \frac{3}{2} \\ 2(x) &= 6(3) \\ 2x &= 18 \\ \frac{2x}{2} &= \frac{18}{2} \\ x &= 9 \end{aligned}$$

EXAMPLE #2

$$\begin{aligned} \frac{3x + 5}{4} &= \frac{7}{2} \\ 2(3x + 5) &= 4(7) \\ 6x + 10 &= 28 \\ 6x &= 28 - 10 \\ 6x &= 18 \\ \frac{6x}{6} &= \frac{18}{6} \\ x &= 3 \end{aligned}$$



A. Solve the following.

1. $\frac{x}{2} = \frac{10}{4}$

2. $\frac{x}{6} = \frac{5}{3}$

3. $\frac{4}{x} = \frac{9}{36}$

4. $\frac{x}{7} = \frac{12}{14}$

5. $\frac{6}{x} = \frac{9}{12}$

6. $\frac{9}{10} = \frac{90}{x}$

7. $\frac{4}{3} = \frac{72}{x}$

8. $\frac{x + 12}{3} = \frac{5 + x}{2}$

9. $\frac{3x - 6}{6} = \frac{2x + 3}{5}$

10. $\frac{4x + 6}{2} = \frac{7x + 30}{5}$

11. $\frac{3x + 5}{2} = \frac{7x + 5}{3}$

12. $\frac{3(x - 2)}{4} = \frac{2(x + 3)}{4}$

13. $\frac{12(x + 5)}{2} = \frac{3(x + 20)}{1}$

14. $\frac{5(x + 2)}{3} = \frac{9(x - 2)}{5}$

15. $\frac{6(x + 2)}{4} = \frac{8(x + 1)}{5}$

16. $\frac{3x}{3} = \frac{60}{15}$

17. $\frac{4(x + 2)}{4} = \frac{2(x - 4)}{1}$

18. $\frac{7x - 3}{5} = \frac{5x + 3}{4}$

19. $\frac{7x}{2} = \frac{x}{12}$

20. $\frac{3 + x}{6} = \frac{x - 5}{3}$

4.6 EQUATIONS REVIEW (TYPES I - II - III - IV - RATIO)

A. Solve the following.

- | | | |
|----------------------------------|-----------------------------------|--|
| 1. $\frac{6x}{3} = \frac{5x}{2}$ | 2. $7x - 3 = 18$ | 3. $6(3x - 5) = 4(4x + 10)$ |
| 4. $2x - 3 = 15$ | 5. $7x - 8 = 5x + 12$ | 6. $4x + 2 = 10$ |
| 7. $\frac{5}{7} = \frac{x}{7}$ | 8. $5(2x + 3) = 7(x + 3)$ | 9. $\frac{x}{3} + 4 = 9$ |
| 10. $5x + 3 = 19 + 3x$ | 11. $x + 6 = 10$ | 12. $\frac{4x}{3} = \frac{8}{6}$ |
| 13. $6x - 2 = 3x + 10$ | 14. $\frac{x}{6} = \frac{24}{36}$ | 15. $5(3x - 4) = 7(2x + 6)$ |
| 16. $3x - 5 = 7$ | 17. $\frac{x}{7} - 8 = 10$ | 18. $15x - 4 = 3x + 8$ |
| 19. $\frac{x}{4} = 9$ | 20. $4x + 50 = 1x + 80$ | 21. $2(4x + 6) = 3(6 + 2x)$ |
| 22. $7x + 45 = 2x + 90$ | 23. $3x + 20 = 50$ | 24. $\frac{7(3x + 9)}{2} = \frac{6(2x + 31)}{4}$ |
| 25. $x - 142 = 10$ | 26. $16x + 2 = 9x + 51$ | 27. $2(4x + 8) = 128$ |

B. Extra Practice. Solve the following.

- | | |
|--|---|
| 1. $\frac{x}{5} - 3 = 8$ | 2. $10y - 8 = 5y + 62$ |
| 3. $2x - 18 = 54$ | 4. $\frac{x}{7} = \frac{8}{4}$ |
| 5. $3(2x + 5) = 5(x + 20)$ | 6. $\frac{x - 6}{3} = \frac{2x + 4}{8}$ |
| 7. $x + 3 = 3$ | 8. $18x + 18 = 9x + 36$ |
| 9. $2x - 7 = 1x + 9$ | 10. $4x + 8 = 16$ |
| 11. $\frac{x + 1}{1} = \frac{5(2x + 3)}{11}$ | 12. $8(x - 2) = 5(x + 1)$ |
| 13. $3x + 2x - 4x + 5x = 18$ | 14. $\frac{x}{7} - 10 = 21$ |
| 15. $\frac{8}{10} = \frac{36}{x}$ | 16. $4(2x - 8) = 3(2x + 6)$ |
| 17. $12x + 8 = 7x + 48$ | 18. $\frac{3(2x + 5)}{12} = \frac{75}{4}$ |
| 19. $\frac{3}{m} = \frac{9}{12}$ | 20. $2a + 3 = 15$ |
| 21. $3x - 8 = 10$ | 22. $\frac{y}{5} + 8 = 20$ |
| 23. $4x = 1000$ | 24. $3x + 2x = 5$ |
| 25. $\frac{2(3x - 9)}{1} = \frac{5(x + 8)}{1}$ | 26. $17x - 15x + 26 = 30 - 2$ |