CHAPTER 6 - FACTORS AND MULTIPLES

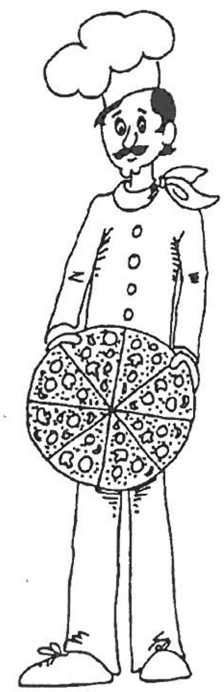
6.1 RULES FOR DIVISIBILITY

Before actually doing a division question, it is possible to tell if certain smaller numbers are factors of, or can divide evenly into a larger number.

We can tell if 2, 3, 4, 5, or 10 are factors of, or can divide evenly into a certain number by knowing and applying the rules listed below.

RULES FOR DIVISIBILITY

A # can be divided by "*n*"	RULE	EXAMPLE
2	If the number is even: (i.e., ends in a 0, 2, 4, 6 or 8)	Can 526 be divided by 2? **Yes,** because 26 is an even number.
3	If the sum of the digits of the number is divisible by 3	Can 534 be divided by 3? **Yes,** because 5 + 3 + 4 is 12 and 3 divides into twelve.
4	If the last two digits of the number are divisible by 4	Can 728 be divided by 4? **Yes,** because the last two digits can be divided by 4.
5	If the last digit of the number is either a 5 or a 0	Can 685 be divided by 5? **Yes,** because the number ends in a 5.
10	If the last digit of the number is a 0 (zero)	Can 9870 be divided by 10? **Yes,** because the number ends in a zero.



A. Answer the following without doing the actual division.

1. Is 2 a factor of 64? ____________________
2. Can 3 divide into 627? ____________________
3. Is 456 divisible by 5? ____________________
4. Can we divide 4 into 6558? ____________________
5. Are 2 and 3 both factors of 1560? ____________________
6. Is the number 4 a factor of 315? ____________________
7. Can we divide 1119 by 3? ____________________
8. List the smallest number we must add to 73 to make it a factor of 3? _____
9. Can 552 be divided by 3? ____________________
10. List three factors of 12 ____________________

6.2 FACTORS

If a number 'f' divides evenly into another number 'm', we say that the number 'f' is a **factor** of 'm'.

The smallest factor of all numbers is the number '1' (one), and the largest factor of all numbers is the number itself.

Listing all the factors of a particular number is the same as finding all the numbers that divide evenly into this particular number. The examples below show all the factors of two numbers.

EXAMPLES #1

List all the factors of 18.
Since all the possible ways of writing 18 in factored form are: **1 × 18, 2 × 9,** and **3 × 6,** all the factors of **18** are:
(1, 2, 3, 6, 9, 18)

EXAMPLE #2

List all the factors of 50
Since all the possible ways of writing 50 in factored form are: **1 × 50, 2 × 25,** and **5 × 10,** all the factors of **50** are:
(1, 2, 5, 10, 25, 50)

(As you can see, every number has only a certain number of factors)

A. Write all the factors for each.

1. 24 =
2. 6 =
3. 27 =
4. 12 =
5. 10 =
6. 15 =
7. 72 =
8. 102 =
9. 144 =
10. 54 =

B. Find the missing factor or divisor ('x') in each question below.

1. $(5)(x) = 50$
2. $(2)(x) = 60$
3. $\frac{27}{x} = 3$
4. $\frac{36}{x} = 9$
5. $(15)(x) = 45$
6. $28 = (7)(x)$
7. $(16)(x) = 16$
8. $98 \div 7 = x$
9. $49 = 1(x)(x)$

C. There are 36 marbles in a bag. To what number of different people could you pass out the marbles evenly with no marbles left over?

D. Find a number or numbers less than 15 that has 6 factors.

6.3 MULTIPLES

A number 'm' is a **multiple** of another number 'f', if the number 'f' divides evenly into the number 'm', or if the number 'f' is a factor of the number 'm' as shown in the examples below.

EXAMPLE #1

List the set of multiples of 5:
(5, 10, 15, 20, 25, 30, • • •)

EXAMPLE #2

List the set of multiples of 7:
(7, 14, 21, 28, 35, 42, • • •)

A. Write the set of the first 5 multiples for each.

1. 2 =
2. 9 =
3. 6 =
4. 8 =
5. 3 =
6. 25 =
7. 13 =
8. 7 =
9. 15 =
10. 11 =

B. Circle the number in the sets which are multiples of the given number.

1. **3** - (3, 9, 11, 15, 36, 51, 66, 392, 408, 20, 726)
2. **10** - (15, 20, 25, 35, 50, 58, 90)
3. **9** - (27, 45, 248, 669, 450)
4. **4** - (12, 16, 28, 40, 50, 60, 108, 244)
5. **25** - (60, 75, 175, 400, 455)
6. **12** - (36, 44, 56, 98, 132)
7. **15** - (20, 25, 30, 60, 75, 85, 100, 200, 300)
8. **5** - (5, 10, 15, 25, 40, 95, 104, 204, 305)
9. **14** - (28, 56, 74, 80, 94, 140, 280, 294)
10. **16** - (32, 64, 132, 160, 164, 204)

C. A grocery store sells oranges in bags of 8. Can you buy 116 oranges? Can you buy 108 oranges? Can you buy 168 oranges?

D. What is the maximum number of quarters you could get if you cashed a five dollar bill?

E. A pop distributer sells a case of pop containing 24 bottles. How many cartons with 6 bottles in each carton are in a case of pop?

F. Explain the relationship between factors of a number and multiples of a number.

6.4 PRIME AND COMPOSITE NUMBERS

PRIME NUMBERS

A **Prime Number** is a number that has only two factors, the number **1** and the number itself. Examples of prime numbers are: **2, 3, 23, 47, 17, 37**. All of these numbers have only two factors.

COMPOSITE NUMBERS

A **Composite Number** is any number that has more than two factors. Examples of composite numbers are: **4, 25, 35, and 62**. All of these numbers have more than two factors.

(* The number '1' is considered to be neither prime nor composite.)

A. List a set of factors for each number below and then state whether this number is a prime number or a composite number.

1. 14 =
2. 27 =
3. 29 =
4. 49 =
5. 121 =
6. 79 =
7. 26 =
8. 8 =
9. 55 =
10. 10 =
11. 24 =
12. 19 =
13. 40 =
14. 99 =
15. 80 =
16. 57 =
17. 45 =
18. 51 =
19. 91 =
20. 2 =

B. Answer the following.

1. List all the composite numbers less than 17.

2. List all the numbers less than 13 that are prime numbers.

3. List the prime numbers less than 20 but greater than 10.

4. List all the composite numbers less than 16 that have 4 factors.

5. List the prime numbers less than 9.

6.5 RELATIVELY PRIME NUMBERS

Two numbers are **relatively prime** if the only common factor they share is the number '1'. Even though both numbers may be composite, they will be considered relatively prime because they have no common factors other than '1', as shown in the example below.

EXAMPLE: Are 26 and 35 relatively prime?

The factors of 26 are: (1, 2, 13, 26)
The factors of 35 are: (1, 5, 7, 35)

Since the only common factor that these two numbers have in common is the number '1', we say that 26 and 35 are **relatively prime**.

A. Determine which of the following pairs of numbers are relatively prime.

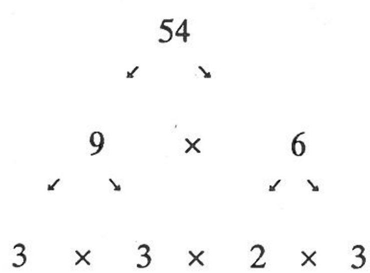
1. 19 and 18
2. 25 and 49
3. 16 and 48
4. 15 and 36
5. 15 and 40
6. 368 and 596
7. 12 and 11
8. 69 and 153
9. 65 and 96
10. 19 and 57

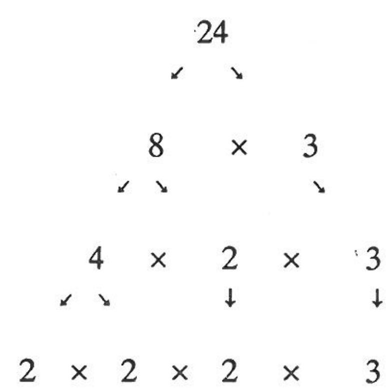
6.6 PRIME FACTORIZATION OF A NUMBER

When we write numbers as a product of prime numbers, we usually use one of two methods: the **factor tree method,** or the **repeated division method.** In both cases when we are finished, we will have a set of factors for a particular number that when multiplied will give you this number.

FACTOR TREE METHOD

EXAMPLES: Prime factor the numbers 54 and 24 using the **factor tree method.**





* The bottom row of numbers must all be **prime numbers** when we complete our factoring.

REPEATED DIVISION METHOD

EXAMPLE #1

Prime factor 84 using the Repeated Division Method.

$$\begin{array}{r|l} 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ 7 & 7 \\ & 1 \end{array}$$

$\therefore 84 = 2 \times 2 \times 3 \times 7$

EXAMPLE #2

Prime factor 210 using the Repeated Division Method.

$$\begin{array}{r|l} 3 & 210 \\ 7 & 70 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$\therefore 210 = 3 \times 7 \times 2 \times 5$

* All numbers you chose to use as factors must be **prime numbers.**

A. Prime factor the following using the **Factor Tree Method.**

1. 24
2. 36
3. 21
4. 54
5. 360
6. 12
7. 48
8. 120
9. 70

B. Prime factor the following using the **Repeated Division Method.**

1. 42
2. 28
3. 30
4. 100
5. 72
6. 84
7. 88
8. 60
9. 990

6.7 GREATEST COMMON FACTOR (GCF)

The **Greatest Common Factor** of two numbers is the largest number that will divide evenly into both numbers. To find the **GCF** we can use either of the two methods shown below.

PRIME FACTOR METHOD

EXAMPLE #1: Find the Greatest Common Factor of 24 and 60.

$$
\begin{array}{rcccccccccl}
24 & = & 2 & \times & 2 & \times & 2 & \times & 3 & & \\
60 & = & 2 & \times & 2 & \times & & & 3 & \times & 5 \\
\hline
\text{GCF} & = & 2 & \times & 2 & \times & & & 3 & & = 12
\end{array}
$$

(Prime factor each number and list the same factors under each other.)

Since 2, another 2 and the 3 are common factors of both, we multiply these numbers together to get **12**, which is the GCF of 24 and 60.

EXAMPLE #2: Find the Greatest Common Factor of 42, 56 and 140.

$$
\begin{array}{rcccccccccl}
42 & = & 2 & \times & 3 & \times & 7 & & & & \\
56 & = & 2 & \times & & & 7 & \times & 2 & \times & 2 \\
140 & = & 2 & \times & & & 7 & \times & 2 & \times & \times\ 5 \\
\hline
\text{GCF} & = & 2 & \times & & & 7 & & & & = 14
\end{array}
$$

Since the 2 and the 7 are the only common factors of all three numbers the GCF is $2 \times 7 = \mathbf{14}$.

REPEATED DIVISION METHOD

EXAMPLE #1: Find the Greatest Common Factor of 60 and 84.

$$
\begin{array}{r|cc}
2 & 60 & 84 \\
\hline
2 & 30 & 42 \\
\hline
3 & 15 & 21 \\
\hline
 & 5 & 7
\end{array}
$$

(Any factor we chose on the left must be a prime number and must also be a common factor of both numbers.)

∴ the GCF is $2 \times 2 \times 3 = \mathbf{12}$. *(We multiply the numbers on the left together.)*

EXAMPLE #2: Find the Greatest Common Factor of 60, 150 and 135.

$$
\begin{array}{r|ccc}
5 & 60 & 150 & 135 \\
\hline
3 & 12 & 30 & 27 \\
\hline
 & 4 & 10 & 9
\end{array}
$$

(When no other number divides into all 3 bottom numbers, we stop.)

∴ the GCF of 60, 150 and 135 is $5 \times 3 = \mathbf{15}$.

A. Using the **Prime Factor Method**, find the GCF for each of the following.

1. 70 and 35
2. 36 and 54
3. 76 and 66
4. 8, 20 and 14
5. 28 and 42
6. 75, 100 and 125
7. 81 and 75
8. 12, 18 and 20
9. 30 and 32
10. 8, 12 and 15

B. Using the **Repeated Division Method,** find the GCF for each of the following.

1. 14, 21 and 35
2. 10, 15 and 25
3. 100, 50 and 75
4. 60 and 84
5. 500, 200, 300 and 900
6. 18, 27, 36 and 25
7. 72, 20, 28, and 48
8. 36, 90 and 210
9. 24, 48, 132 and 240
10. 74, 148 and 296

C. Using either of the two methods (Prime Factor or Repeated Division), find the GCF of the following sets of numbers.

1. 9 and 12
2. 90 and 36
3. 40 and 90
4. 102 and 48
5. 36 and 180
6. 32, 12 and 20
7. 36, 90 and 144
8. 48, 84 and 72
9. 30, 12 and 18
10. 24 and 120

6.8 LOWEST COMMON MULTIPLE (LCM)

The **Lowest Common Multiple** of two or more numbers is the *'smallest'* number that two or more numbers will divide evenly into. Two methods used for determining the LCM are shown below.

PRIME FACTOR METHOD

EXAMPLE: Find the lowest common multiple of 24, 60 and 32.

$$\begin{array}{rcl} 24 & = & 2 \times 2 \times 2 \times 3 \\ 60 & = & 2 \times 2 \times 3 \times 5 \\ 32 & = & 2 \times 2 \times 2 \times \times 2 \times 2 \\ \hline \text{LCM} & = & 2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 2 = 480 \end{array}$$

(Prime factor and list the same factors under each other.)

(To find the LCM we list ALL the factors or numbers and multiply them together.)

REPEATED DIVISION METHOD

EXAMPLE: Find the Lowest Common Multiple of 24, 30 and 36.

$$\begin{array}{r|ccc} 2 & 24 & 30 & 36 \\ \hline 3 & 12 & 15 & 18 \\ \hline 2 & 4 & 5 & 6 \\ \hline & 2 & 5 & 3 \end{array} \leftarrow$$

At this point we now must find a number that will divide into at least two numbers. If a number is not divisible by the selected factor, rewrite it below itself.

$\therefore$ the LCM of 24, 30 and 36 = $2 \times 3 \times 2 \times 2 \times 5 \times 3$ = **360**

A. Find the LCM of the following sets of numbers using the **Prime Factor Method**.

1. 4, 6 and 10
2. 6 and 15
3. 28 and 21
4. 12 and 15
5. 14 and 28
6. 8, 10 and 12
7. 35, 14 and 21
8. 12 and 18
9. 4, 10 and 12
10. 15, 9 and 25

B. Find the LCM for the following sets of numbers using the **Repeated Division Method.**

1. 12 and 28
2. 15, 6 and 24
3. 24, 21 and 9
4. 40, 24 and 8
5. 45 and 60
6. 22 and 33
7. 12, 15 and 24
8. 10, 20, 30 and 40
9. 21, 33 and 90
10. 8, 11, 32 and 60

C. Using either the Prime Factor Method or the Repeated Division Method, find the **LCM** for the following.

1. 12 and 24
2. 8 and 10
3. 40 and 50
4. 6 and 9
5. 30, 40 and 50
6. 12, 16 and 20
7. 20, 12 and 8
8. 16, 18 and 14
9. 6, 8 and 4
10. 20, 25 and 30

6.9 FACTORS AND MULTIPLES REVIEW

A. Prime factor the following numbers.

1. 72
2. 250
3. 45
4. 720

B. Find the missing factors in the following.

1. $637 = 7 \times 7 \times \square$
2. $98 = 7 \times \square \times 2$
3. $2401 = \square \times 7 \times 7 \times 7$
4. $270 = 3 \times 5 \times 2 \times \square \times \square$

C. Determine which of the following pairs of numbers are relatively prime.

1. 35 and 26
2. 72 and 63
3. 12 and 15
4. 32 and 34
5. 72 and 61
6. 372 and 153
7. 560 and 425
8. 101 and 606
9. 99 and 100

D. Find <u>**both**</u> the **GCF** and **LCM** for each of the following.

1. 60 and 72
2. 32 and 64
3. 16 and 20
4. 105 and 180
5. 18, 33 and 45
6. 8, 10 and 12
7. 14, 21 and 15
8. 77 and 98
9. 6, 9 and 27

E. Complete the alphabet by placing the missing letters above or below the given line. Each question has a different pattern.)

1. A B D
C E F G H

2. A E F H I
B C D G

3. B C E G
A
D F H I J

F. Solve the following using your knowledge of factors and multiples.

1. Eggs are packed 18 eggs to a large carton. Can an egg producer pack 864 eggs using full cartons?

2. Gary and Leonard are buying bags of marbles where each bag has the same number of marbles. The total number of marbles in the bags Gary buys is 15, and the number of marbles in the bags that Leonard buys is 24. What are the different possible number of marble(s) in each bag?

3. Wieners come in packages of 10 and hot dog buns come in packages of 12. What is the least number of both you must buy in order to have the same number of wieners and hot dog buns?