CHAPTER 2 - NUMBER EXPRESSIONS

2.1 ORDER OF OPERATIONS (BEDMAS)

Centuries ago, it was decided by mathematicians from all over the world that all nations must follow the same rules when calculating number questions. This was decided so that all answers for the same questions from different countries would be the same. The rules, or **ORDER OF OPERATIONS** that these mathematicians decided to follow are as follows:

1. Do the calculations appearing in the **parentheses** first.
2. Do any calculations that involve **exponents** next.
3. Calculate the **multiplication** and **division** next.
4. Do the **addition** and **subtraction** last.

One method we use to remember the order in which we do mathematical operations is to use the acronym (first letter of each word) **BEDMAS** as shown in the chart below.

B	E	D	M	A	S
BRACKETS	EXPONENTS	DIVIDE	MULTIPLY	ADD	SUBTRACT

The examples below show the procedure followed solving questions involving multiple operations.

1. If **addition** and **subtraction** occur in the same expression, perform the operations in the order in which they occur.

EXAMPLE: $\underline{11 + 4} - 5 + 2$
$\underline{15 - 5} + 2$
$10 + 2 = 12$

2. If **multiplication** and **addition** or **subtraction** occur together, perform the operation of multiplication first.

EXAMPLE: $16 + \underline{3 \times 4} - \underline{2 \times 3} + 3$
$\underline{16 + 12} - 6 + 3$
$\underline{28 - 6} + 3$
$22 + 3 = 25$

3. If **division** and **addition** or **subtraction** occur together, perform the operation of division first.

EXAMPLE: $9 + \underline{16 \div 4} - 2$
$\underline{9 + 4} - 2$
$13 - 2 = 11$

4. If **multiplication, division, addition** or **subtraction** occur together, first perform the multiplication and division in the order they occur and then perform the **addition** and **subtraction.**

EXAMPLE: $\underline{6 \times 4} \div 8 + 5$
$\underline{24 \div 8} + 5$
$3 + 5 = 8$

5. If **brackets** occur in a number expression, perform whatever operation is enclosed in the brackets first, then follow the rules above.

EXAMPLE: $12 \div \underline{(9 - 5)} \times 2$
$\underline{12 \div 4} \times 2$
$3 \times 2 = 6$

6. If a **line** such as we use in division or fractions occurs in a number expression, evaluate the numerator (top part) and the denominator (bottom part) before dividing.

EXAMPLE: $\dfrac{7 \times 4 + 2}{2 + 3 \div 3} = \dfrac{30}{3} = 10$

A. Calculate each of the following using the rules for order of operations.

1. $19 + 7 - 6 + 10$
2. $36 \div (3 \times 3) + 4$
3. $36 \div 12 + 15 - 8$
4. $23 + 15 - 9 \times 3$
5. $144 \div 12 \times 3$
6. $8 - 4 - 2 + 6$
7. $(6 - 5) \times 15 - 2$
8. $18 - 2 \times 9 + 15$
9. $25 + 37 - 8 - 15$
10. $7 \times (9 - 5) + 6$
11. $3 + 0 \times 5 + 14$
12. $46 \times 2 - 15 \times 2$
13. $6 \times 5 \div 2 \times 5$
14. $5 + 6 + 17 - 3$
15. $(60 - 6) \div 2$
16. $49 \div (11 - 4)$
17. $28 \div 4 + 3 \times 8 - 6$
18. $7 + 96 \div 3$
19. $100 \div 4 - 25$
20. $93 - 3 \times 30 - 3$
21. $3 \times (2 + 7) - 3$
22. $15 + 21 \div 3 - 3$
23. $19 + 36 - 15 \times 3$
24. $19 \times (6 - 6)$
25. $(9 \times 7) - (7 \times 9)$
26. $39 \div (10 + 3) - 3$
27. $16 \div 4 \times 3 - 5$
28. $9 \div 3 \times 3 + 1$
29. $(27 \div 3) \times 6$
30. $5 \times 4 \div 2$
31. $48 \div (6 - 2) - 8$
32. $36 \div 6 \times 2 + 4$
33. $30 \div 15 \times 8 + 10$
34. $5 + 3 \times 2 - 6$
35. $15 - 3 \times 4 + 3 \times 1$
36. $5 + 3 \times 2 - 4$
37. $10 + 3 \times 14 - 6$
38. $64 \div (16 \div 2)$
39. $8 \times 9 - 15 \div 5$
40. $49 + 8 \div 8 + 49$
41. $6 \times 3 + 5 \times 6$
42. $7 - 2 + 18 - 3$
43. $12 + 35 - 30 \div 6$
44. $8 - 3 + 3 - 8$
45. $56 \div 8 \times 4 \div 2$
46. $10 - 1 + 10 - 1$
47. $(4 + 14) \div (18 \div 2)$
48. $63 \div (7 - 4) + 99$

B. Calculate each of the following,

1. $\frac{7 + 8 + 9}{5 + 2 + 5}$

2. $\frac{20 + 4 - 8}{12 - 4}$

3. $\frac{37 - 15 + 8}{16 + 9 - 15}$

4. $\frac{5 \times 4 + 4}{3 \times 3 - 3}$

5. $\frac{50 \div 2 + 15}{4 + 32 \div 2}$

6. $\frac{(6 + 4) \times (3 + 2)}{(3 + 2) \times (4 + 1)}$

7. $\frac{17 - 9 + 4}{(20 + 8) \div 7}$

8. $\frac{9 + 24}{17 - 2 \times 3}$

9. $\frac{4 \times (6 - 3) \div 6}{(19 - 5) \div 7}$

10. $\frac{15 - 2 \times 3}{3 \times 3}$

11. $\frac{58 - 18 - 40}{7 \times 6 + 4}$

12. $\frac{48 + 5 - 19}{2 \times 8 + 1}$

13. $\frac{3 \times (15 - 3)}{6 \times (11 - 8)}$

14. $\frac{12 \times (7 + 5)}{64 \div (7 + 1)}$

15. $\frac{43 \times (3 - 1) + 6}{6 + 43 \times (3 - 1)}$

16. $\frac{24 - (6 + 3)}{3 \times (10 \div 2)}$

17. $\frac{59 - (2 + 26) - 10}{14 + 2 \times 7 - 21}$

18. $\frac{5 \times 9 \div 3 + 17}{64 - (18 + 14)}$

19. $\frac{100 - 10 - 10 - 10}{40 + 10 + 10 + 10}$

20. $\frac{(153 - 100) \times 2}{7 \times 7 + 4}$

21. $\frac{11 + 2 \div (5 - 4)}{35 \div 7 - 4}$

22. $\frac{18 + 9 \times 3 + 15}{27 - 28 \div 4}$

23. $\frac{(21 + 19) \times (15 - 13)}{(8 - 6) \times (27 - 23)}$

24. $\frac{14 + 7 - 9 \times 2}{(15 - 6) \div (14 - 11)}$

25. $\frac{2 \times 9 - 5 + 8 \div 4}{6 \times 5 \div 10}$

26. $\frac{63 \div 7 \times 7}{21 \div 3 \times 3}$

27. $\frac{16 \times (4 \div 4)}{16 \div (4 \div 4)}$

28. $\frac{16 \times 4 \times 4}{16 \div 4 \times 4}$

29. $\frac{14 \times (9 \div 3) + 12}{18 \div (26 - 24)}$

30. $\frac{44 \div (2 \times 5 + 1)}{38 \div (16 + 3)}$

2.2 SUBSTITUTION (SINGLE SYMBOL)

We can use symbols in an expression to take the place of a number and the result (answer) of the expression will depend upon what number we substitute for the symbol. Once we have substituted for the symbol, we use the rules of order of operations (**BEDMAS**) to calculate the value of the expression as shown in the examples below.

EXAMPLE #1

Evaluate the expression below if $\square = 6$

$3 + \square \times 9$

$3 + 6 \times 9$

$3 + 54 = 57$

EXAMPLE #2

Evaluate the expression below if $\square = 7$

$3 + \square \times 9$

$3 + 7 \times 9$

$3 + 63 = 66$

A. Evaluate each expression by substitution.

1. $\square + 3 + \square$, if $\square = 2$

2. $6 \times 3 \times \square \times \square$, if $\square = 3$

3. $5 \times \square - 4$, if $\square = 5$

4. $7 - \square + 15$, if $\square = 4$

5. $8 \times \square - 9 + 3$, if $\square = 3$

6. $(\square \div 3) \times (\square \div 4)$, if $\square = 12$

7. $17 + \square - 2 \times \square$, if $\square = 10$

8. $2 \times \square + 8 \times \square$, if $\square = 8$

9. $3 \times \square + \square \div 4$, if $\square = 16$

10. $\square + 9 - \square$, if $\square = 9$

11. $(3 - \square) \times (\square + 3)$, if $\square = 0$

12. $12 + (\square - 12)$, if $\square = 25$

13. $\dfrac{3 \times \square}{\square - 4}$, if $\square = 7$

14. $\dfrac{\square + 9 + 2 \times \square}{6 \times \square}$, if $\square = 3$

15. $\dfrac{2 \times \square - (\square + 6)}{14 + \square}$, if $\square = 6$

16. $\dfrac{4 + 2 \times \square}{3 \times \square + 1}$, if $\square = 3$

17. $\dfrac{\square - \square + \square}{\square}$, if $\square = 9$

18. $\dfrac{6 + 4 \times \square}{\square + 2}$, if $\square = 0$

19. $\dfrac{\square \times \square \times \square}{\square - 2}$, if $\square = 6$

20. $\dfrac{\square \times \square \times 4}{2 \times \square}$, if $\square = 2$

B. For each question below find the number that makes each sentence correct.

1. $\square + 5 = 12$
2. $\square \times \square = 9$
3. $\square + 12 = 2 \times \square$
4. $\square + \square = 3 \times \square$
5. $2 \times \square - \square = 7$
6. $(\square \times \square) + 5 = 30$
7. $16 + \square + \square = 32$
8. $5 + \square \times 3 = 26$
9. $5 \times \square - \square = 0$
10. $4 \times \square + 3 \times \square = 42$

C. List the set of numbers that makes each sentence correct.

1. $\square < 5$
2. $14 < \square$
3. $5 \times \square = 18$
4. $\square$ is a three digit number
5. $4 \times \square < 12$
6. $\square$ can divide into 15
7. $\square + 18 = 30$
8. $\square > 7$
9. $4 + 7 + \square < 15$
10. $\square > 10$ but < 17

2.3 SUBSTITUTION (DOUBLE SYMBOLS)

Often in mathematics you will be asked to evaluate expressions in which two symbols are used instead of only one. Here, you must take special care to substitute the right variable for the appropriate symbol before you use the order of operations rules (**BEDMAS**). The examples below shows how this substitution is done.

EXAMPLE #1

Evaluate the expression below if $\square = 5$, and $\Diamond = 7$

$3 \times \square + 4 \times \Diamond$
$3 \times \mathbf{5} + 4 \times \mathbf{7}$
$15 + 28 = 43$

EXAMPLE #2

Evaluate the expression below if $\square = 0$, and $\Diamond = 10$

$\Diamond \div 2 - \square \div 8 + \Diamond \times 3$
$\mathbf{10} \div 2 - \mathbf{0} \div 8 + \mathbf{10} \times 3$
$5 - 0 + 30 = 35$

A. Evaluate each of the following by substitution.

1. $\square - \diamond$, if $\square = 5$ and $\diamond = 2$

2. $\square \times \diamond$, if $\square = 7$ and $\diamond = 12$

3. $\diamond + \square - 6$, if $\square = 8$ and $\diamond = 12$

4. $(2 \times \diamond) - (3 \times \square)$, if $\square = 4$ and $\diamond = 11$

5. $2 \times \square + 2 \times \diamond$, if $\square = 10$ and $\diamond = 3$

6. $(\square \div 4) \times (9 - \diamond)$, if $\square = 16$ and $\diamond = 5$

7. $\square \times \diamond - 16$, if $\square = 4$ and $\diamond = 9$

8. $\square \times (\diamond - 2)$, if $\square = 7$ and $\diamond = 6$

9. $\square \times (\diamond - 5)$, if $\square = 8$ and $\diamond = 11$

10. $48 \div (2 \times \square) + \diamond$, if $\square = 6$ and $\diamond = 3$

11. $(5 + \diamond) \times \square$, if $\square = 9$ and $\diamond = 5$

12. $2 + \square + 9 - \diamond$, if $\square = 4$ and $\diamond = 5$

13. $\dfrac{\square + \diamond}{3 + \diamond}$, if $\square = 9$ and $\diamond = 3$

14. $\dfrac{2 \times \square - 3 \times \diamond}{\square - (\diamond + 2)}$, if $\square = 8$ and $\diamond = 5$

15. $\dfrac{\square + \diamond}{\square - \diamond}$, if $\square = 12$ and $\diamond = 6$

16. $\dfrac{12 \times (\square - \diamond)}{\diamond + 9}$, if $\square = 11$ and $\diamond = 7$

17. $\dfrac{\square + 3 \times \diamond}{\diamond}$, if $\square = 4$ and $\diamond = 4$

18. $\dfrac{5 \times \square - 3 \times \diamond}{4 \times (\square - 1)}$, if $\square = 5$ and $\diamond = 3$

19. $\dfrac{\square + \diamond + 2}{\diamond}$, if $\square = 4$ and $\diamond = 2$

20. $\dfrac{\square \times \diamond}{\diamond \times \square}$, if $\square = 5$ and $\diamond = 35$

21. $\dfrac{3 \times \square + 2 \times \diamond}{\diamond}$, if $\square = 7$ and $\diamond = 3$

22. $\dfrac{\diamond + \diamond + \diamond - \square}{\square}$, if $\diamond = 6$ and $\square = 3$

B. For each question below, find the numbers or sets of numbers that will make each statement correct.

1. $\square - \diamond = 7$

2. $3 \times \square + 4 \times \diamond = 24$

3. $\square + \diamond = 8$

4. $2 \times \diamond + \diamond = 13$

5. $2 \times \diamond - 3 \times \diamond = 1$

6. $2 \times \diamond + 3 \times \diamond = 6$

7. $\square + \diamond = 7 + \square$

8. $\diamond = 3 \times \square$

9. $3 \times \diamond = 5 + \square$

10. $(\square + \diamond) \div 2 = 2$