

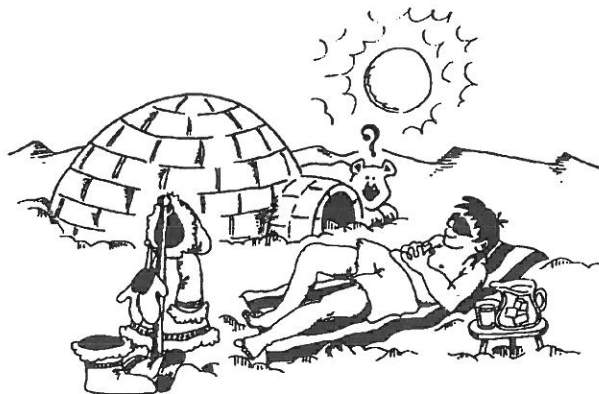
CHAPTER 3 - INTEGERS (I)

3.1 POSITIVE AND NEGATIVE NUMBERS

The set of **integers** is symbolized by the capital letter **I** and is composed of all the **positive numbers**, zero and all the **negative numbers**. The set of integers is:

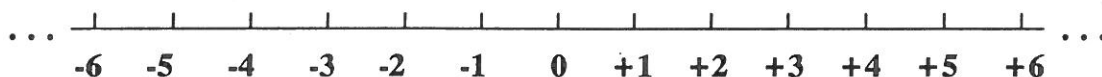
$$I = \{ \dots, -4, -3, -2, -1, 0, +1, +2, +3, +4, \dots \}$$

We read the number **+7** as positive seven and we read the number **-4** as negative four. The number **0** (zero) is neutral, meaning it has no positive or negative sign in front of it.



We use integers extensively in our daily lives. If we hear that the temperature outside is -22°C , we know that we have to dress in warm clothing. A skin diver must know how many metres below sea level (a negative value), he can dive safely and a farmer in Saskatchewan must be able to sell his grain for more than he paid for the seed (a positive value) or he will end up with a negative bank account.

We can graphically represent the set of integers as shown below. We can also use this graph to assist us in determining whether one integer is larger than another and also in adding and subtracting integers.



A. Indicate whether each would result in a positive or negative value.

- | | |
|-------------------------------|--|
| 1. Spending \$15 _____ | 2. Climbing a mountain _____ |
| 3. Falling off a ladder _____ | 4. Going from the 3rd to the 9th floor _____ |
| 5. Earning \$450 _____ | 6. A change in temperature _____ |

B. Write an integer to represent each of the following.

- | | |
|-----------------------------------|--|
| 1. Losing \$10 _____ | 2. Finding a \$100 bill _____ |
| 3. Making a profit of \$250 _____ | 4. A temperature decrease of 3 degrees _____ |
| 5. Scoring 3 goals _____ | 6. Diving down 25 m _____ |

C. Graph each of the following on the given number line.

- | | |
|-----------------------|-------|
| 1. +6, +5, +4, +2, 0 | _____ |
| 2. -6, -5, -4, -2, -1 | _____ |
| 3. -3, +2, 0, +3, -1 | _____ |

D. Write three integers that would appear next in the following sequences.

1. +7, +8, +9, _____ 2. -7, -5, -3, _____
3. -12, -7, -2, _____ 4. 12, 7, 2, _____
5. -9, -8, -6, -3, _____ 6. -1, -5, -10, -16, _____

E. Place a greater than sign (>) or a less than sign (<) between each set of integers to make each statement correct. (The first one is already done for you.)

- | | | | | | |
|------------|------|---------|--------|---------|-----|
| 1. +3 < +6 | 2. 0 | +5 | 3. +15 | -18 | |
| 4. -3 | -6 | 5. -5 | 0 | 6. -13 | -16 |
| 7. -8 | -2 | 8. -1 | -2 | 9. -12 | 0 |
| 10. +3 | -3 | 11. -10 | -12 | 12. 0 | +12 |
| 13. +4 | -2 | 14. +10 | +12 | 15. -15 | -13 |

F. Rearrange each of the following sets of numbers from smallest to largest.

- | | |
|-------------------------|--------------------------------|
| 1. +7, +8, +9, -3 | 2. +6, -6, +8, -8 |
| 3. +3, -8, 0, -5, +6 | 4. 0, -3, -9, -4, -8 |
| 5. -2, -4, -6, +4, -7 | 6. -80, -40, -30, -55 |
| 7. +7, +8, +3, -8, -3 | 8. 0, -6, +4, +9, -3 |
| 9. -7, -9, -19, -1, -18 | 10. -3, -4, -2, +1, -1, -5, -7 |

G. Write out the set of integers described in each.

- The integers greater than +5 but less than +9 _____
- The integers less than -4 but greater than -10 _____
- The integers less than -5 but greater than -8 _____
- The integers greater than -3 but less than -9 _____
- The integers greater than -9 but less than -8 _____

H. Using a number line state where you would end up with each movement.

- | | |
|-------------------------------------|-----------------------------------|
| 1. Start at +8 and move -3 _____ | 2. Start at -8 and move -3 _____ |
| 3. Start at 0 and move -3 _____ | 4. Start at +4 and move -4 _____ |
| 5. Start at -9 and move +5 _____ | 6. Start at -6 and move -3 _____ |
| 7. Start at +8 and move -9 _____ | 8. Start at -3 and move -8 _____ |
| 9. Start at +4 and move +7 _____ | 10. Start at -3 and move 0 _____ |
| 11. Start at -14 and move +23 _____ | 12. Start at -2 and move -2 _____ |
| 13. Start at +5 and move +6 _____ | 14. Start at -5 and move -9 _____ |
| 15. Start at -3 and move -19 _____ | 16. Start at -4 and move +9 _____ |

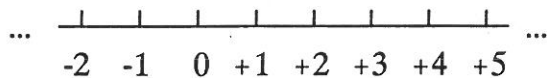
3.2 ADDITION OF INTEGERS

Integers can be added by using one of the two methods described below.

METHOD #1: A **number line** can be used to do addition by moving left or right on this number line. We position ourselves on the first integer given in the question and the next integer tells us whether we travel left or right on the number line. A **positive** integer means we move to the **right**, and a **negative** integer means we move to the **left** on the number line. The examples below show how this is done.

EXAMPLE #1

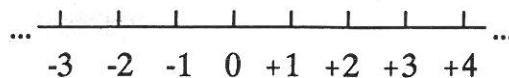
$$(+4) + (-5) = ?$$



We start at +4 and move 5 spaces to the left. $\therefore (+4) + (-5) = -1$

EXAMPLE #2

$$(-2) + (+5) = ?$$



We start at -2 and move 5 spaces to the right. $\therefore (-2) + (+5) = +3$

METHOD #2: The second method of addition involves learning **Two Rules** for addition which are:

- | | |
|--|---|
| 1) If the signs of the integers are the same , add the numbers and keep the same sign. | 2) If the signs of the integers are different , subtract the numbers and take the sign the numerically larger number. |
|--|---|

EXAMPLE #1

$$(+3) + (+5) = +8$$

EXAMPLE #2

$$(-7) + (-5) = -12$$

EXAMPLE #1

$$(-9) + (+4) = -5$$

EXAMPLE #2

$$(+8) + (-3) = +5$$

A. Starting at zero, translate each of the following situations into an expression involving integers and find the integer that represents the overall change. (The first one is already completed.)

1. 8 to the right, five to the left $(0) + (+8) + (-5) = +3$
2. 9 to the left, 8 to the right _____
3. 3 to the left, 5 to the right _____
4. 6 to the right, 6 to the left _____
5. 9 to the left, 7 to the right _____
6. 1 to the right, 8 to the left, 4 to the right _____
7. 5 to the left, 4 to the left, 3 to the left _____
8. 6 to the right, 3 to the right, 14 to the left _____
9. 2 to the left, 9 to the left, 7 to the right _____
10. 15 to the right, 8 to the left, 8 to the right _____
11. 23 to the right, 18 to the left, 4 to the right _____
12. 5 to the left, 5 to the left, 5 to the left _____

B. Translate each of the following situations into an expression involving integers and find the integer that represents the overall change.

1. The temperature starts at -15°C , drops 10°C , rises 5°C and rises 8°C .
2. A person starts with \$50, earns \$12, spends \$15, earns \$18 and spends \$22.
3. Carol is 62 kg. She gains 5 kg, loses 12 kg, gains 9 kg and loses 14 kg.
4. The temperature starts at 22°C , drops 9° , drops 5° , rises 8° and drops 12° .
5. A submarine starts at sea level, dives down 125 m, dives another 72 m, and rises 42 m.
6. A certain stock starts at 320 points, gains 15 points, drops 35 points, and drops 18 points.
7. An elevator starts on the 7th floor, descends 5 floors and ascends 9 floors.
8. A helium balloon is released, gains 73 m, drops 42 m, gains 27 m, and drops 58 m.
9. An airplane takes off, gains 950 m, gains another 270 m, drops 35 m and drops 43 m.
10. The temperature starts at 28°C , drops 14°C , rises 12°C , drops 5°C and gains 7°C .

C. Find the result of these sums.

$1. (+7) + (+3) =$

$2. (-6) + (-3) =$

$3. (-8) + (-6) =$

$4. (+6) + (-3) =$

$5. (-7) + (-8) =$

$6. (-4) + (+2) =$

$7. (+4) + (-2) =$

$8. (-3) + (-14) =$

$9. (-3) + (+5) =$

$10. (+8) + (-6) =$

$11. (+5) + (-8) =$

$12. (-8) + (+2) =$

$13. (+7) + (-9) =$

$14. (-8) + (+6) =$

$15. (-7) + (-9) =$

D. Add the following integers.

$(+10) + (+3) =$

$(+5) + (-3) =$

$(0) + (-7) =$

$(+12) + (-4) =$

$(+6) + (-7) =$

$(+7) + (-2) =$

$(+9) + (+10) =$

$(+5) + (-3) =$

$(-6) + (+3) =$

$(+3) + (-4) =$

$(+5) + (-9) =$

$(+5) + (+5) =$

$(-7) + (+4) =$

$(+6) + (+1) =$

$(+4) + (-4) =$

$(+2) + (+7) =$

$(+9) + (0) =$

$(+7) + (-3) =$

$(-7) + (-7) =$

$(+5) + (-9) =$

$(-6) + (-2) =$

$(+3) + (+7) =$

$(-4) + (-9) =$

$(-2) + (-5) =$

$(-5) + (-5) =$

$(-9) + (-7) =$

$(-4) + (-8) =$

$(-9) + (-8) =$

E. Add the following integers.

$(+13) + (-12) =$

$(+18) + (-15) =$

$(+13) + (+19) =$

$(-17) + (-23) =$

$(+14) + (-14) =$

$(+18) + (+18) =$

$(-16) + (-16) =$

$(-17) + (-13) =$

$(-34) + (-12) =$

$(+12) + (+17) =$

$(-19) + (-15) =$

$(-23) + (-25) =$

$(+29) + (+34) =$

$(+34) + (+21) =$

$(+23) + (+35) =$

$(-45) + (-34) =$

$(-78) + (-21) =$

$(-32) + (-21) =$

$(-55) + (-55) =$

$(-14) + (-84) =$

$(+53) + (+49) =$

$(-46) + (-46) =$

$(+75) + (-75) =$

$(-90) + (-90) =$

$(-14) + (-10) =$

$(-23) + (-41) =$

$(-16) + (-18) =$

$(+34) + (-43) =$

$(-89) + (-33) =$

$(-52) + (+52) =$

$(+44) + (-43) =$

$(-56) + (+34) =$

$(-34) + (+67) =$

$(-23) + (-23) =$

$(+54) + (+59) =$

$(+35) + (-35) =$

F. Complete the following addition chart. A few of the sums are already in place.

+	+3	-4	0	+5	-2	+7	-8	-1
-7				-2	-9			
-3								
+2	+5							
-9				-4				
-8		-12						

G. Calculate each of the following.

1. $(-6) + (-3) + (-4)$
2. $(+4) + (-3) + (-8)$
3. $(-4) + (-3) + (+2)$
4. $(-8) + (+4) + (+3)$
5. $(+7) + (-2) + (+3)$
6. $(+6) + (-3) + (+3)$
7. $(-7) + (-3) + (-4)$
8. $(+5) + (0) + (-6)$
9. $(-8) + (-7) + (+4)$
10. $(-3) + (-4) + (-4)$
11. $(-3) + (-8) + (-5)$
12. $(0) + (-3) + (-5)$
13. $(+3) + (+6) + (+5)$
14. $(-5) + (+5) + (0)$
15. $(+9) + (-3) + (-5)$
16. $(+4) + (+3) + (-3)$
17. $(+3) + (-4) + (-6)$
18. $(+4) + (-5) + (-5)$
19. $(-3) + (-8) + (-5)$
20. $(+8) + (-4) + (-3)$
21. $(-6) + (-3) + (-5)$
22. $(+4) + (-5) + (-3)$
23. $(+9) + (-6) + (+2)$
24. $(+9) + (-4) + (-3)$
25. $(+5) + (+5) + (+5)$
26. $(-1) + (-1) + (-2)$
27. $(+9) + (+4) + (-3)$
28. $(-3) + (-8) + (+4)$
29. $(-1) + (-2) + (-3)$
30. $(+2) + (-4) + (-6)$

H. Fill in the blanks.

1. A positive integer plus a positive integer = _____
2. A negative integer plus a negative integer = _____
3. A negative integer plus a positive integer = _____

I. The coldest place in the world is at Plateau Station in the Antarctic where the annual average temperature is -58°C . The hottest place in the world is in Dalol Danakil Depression in Ethiopia where the annual average temperature is $+35^{\circ}\text{C}$. How much hotter is the average temperature in Dalol than it is in Plateau Station?

3.3 INTEGER REVIEW

A. Add each of the following

1. $(-15) + (+12)$
2. $(-13) + (-18) + (-16)$
3. $(+65) + (-83) + (+42)$
4. $(-21) + (-35)$
5. $(-42) + (-35) + (+62)$
6. $(-52) + (-52) + (-36)$
7. $(+32) + (+64)$
8. $(-27) + (+53) + (+27)$
9. $(-85) + (+99) + (-17)$
10. $(-52) + (-83)$
11. $(-41) + (-36) + (+62)$
12. $(+27) + (+27) + (+27)$
13. $(-61) + (-35)$
14. $(+53) + (-78) + (+26)$
15. $(-18) + (-19) + (-20)$
16. $(+18) + (-25)$
17. $(+9) + (+15) + (-102)$
18. $(+88) + (+99) + (+111)$
19. $(-53) + (+35)$
20. $(+53) + (-47) + (-38)$
21. $(-62) + (-43) + (-82)$
22. $(-7) + (+18) + (-7)$
23. $(-29) + (-14) + (+100)$
24. $(-42) + (+25) + (-104)$
25. $(-6) + (+14) + (-8)$
26. $(+36) + (-103) + (-36)$
27. $(-37) + (-7) + (+65)$
28. $(+47) + (-46) + (+45)$
29. $(-134) + (-56) + (+34)$
30. $(+64) + (-83) + (-75)$
31. $(+67) + (+56) + (+748)$
32. $(+456) + (-465) + (+102)$
33. $(-76) + (-77) + (-78)$
34. $(+56) + (-83) + (-57)$
35. $(-90) + (-37) + (+99)$
36. $(+98) + (-65) + (-34)$
37. $(+153) + (-7) + (-632)$
38. $(-65) + (-73) + (+56)$
39. $(-64) + (-43) + (-67)$
40. $(+45) + (-86) + (-123)$
41. $(0) + (-23) + (-15) + (-2)$
42. $(+876) + (-876) + (-203)$
43. $(-1) + (-2) + (-3) + (-4)$
44. $(+76) + (-62) + (-34)$
45. $(-3) + (-67) + (-45) + (-51)$
46. $(+34) + (-341) + (+3410)$
47. $(+45) + (+279) + (-431)$
48. $(-53) + (-87) + (+339)$